ON THE POSSIBILITY OF THE OMPR EFFECT IN SPACES WITH FINSLER GEOMETRY. PART II

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As a continuation of the ideas in [1], we determine a new solution for Einstein equations in vacuum for linearly approximable anisotropic perturbations of flat Minkowski and Berwald-Moor Finslerian metric. Also, we determine the effective expressions for geodesics and eikonal for small anisotropic perturbations of Minkowski and Berwald-Moor metrics and the changes of the OMPR conditions for the two models. This could in principle provide the possibility to study the anisotropic properties of space-time in our galaxy.

1 Introduction

This paper is the continuation of [1] in which the motivation and the physical background of this research were given in detail. To remind it briefly, we should mention the following. The astrophysical data collected during the last decade contradicts the GRT expectations for several specific cases and demands to modify the expression for the Einstein-Hilbert action which is the base for the popular cosmological models. Several such attempts appeared to be unsatisfactory for various reasons, and in [1] the modification accounting for the possible anisotropy of space-time was suggested. The consequences of such suggestion are meaningful, therefore, there must be an experimental test able to show if this idea could be applied to the physical world. The corresponding effect is the optic-metrical parametric resonance (OMPR) exploiting the metrical properties of space-time, particularly, the existence and properties of the gravitational waves. The theory of OMPR can be found in [2], [3], [4], [5], the first experimental evidence of its existence are given in [6].

The goal of [1] was to adjust the ideas underlying the theory of the OMPR effect to the case when the geometry used for the description of the space-time is not Riemannian but the Finslerian one. This included the modifications of the Einstein equations, of the eikonal equation and of the geodesic equations. In order to do this the needed mathematical formalism was introduced, and the Einstein equations in vacuum, [7], for the linearized h-v model

$$G = (\gamma_{ij}(y) + \varepsilon_{ij}(x,y))dx^i \otimes dx^j + v_{ab}(x,y)\delta y^a \otimes \delta y^b$$

(where $\gamma_{ij} = \gamma_{ij}(y)$ is a locally Minkowski metric, $\varepsilon_{ij} = \varepsilon_{ij}(x,y)$ and $v_{ab} = v_{ab}(x,y)$ are small anisotropic perturbations), became:

$$\begin{cases} R_{ij} - \dfrac{1}{2}R\gamma_{ij} = \dfrac{1}{2}S(\gamma_{ij} + \varepsilon_{ij}) \\ (\delta^i_s\delta^l_j - \gamma^{il}\gamma_{sj})\Gamma^s{}_{li\cdot b} = 0 \\ S_{ab} - \dfrac{1}{2}(r+S)v_{ab} = 0 \end{cases} . \tag{1}$$

Here, $\Gamma^i{}_{jk}$ are the usual Christoffel symbols for $g_{ij} = \gamma_{ij} + \varepsilon_{ij}$, R_{ij} is the corresponding Ricci tensor, and S_{ab} is the Ricci tensor corresponding to the "vertical" part v_{ab} of the metric structure G.

We ascertained that they preserve the form of the wave equation $\Box\varepsilon_{ij} = 0$ for the direction dependent metric, while the solution $\varepsilon_{jh} = Re(a_{jh}(y)e^{ik_m(y)x^m})$ appeared to be expectedly different from the regular plane wave even for the simplest case: its amplitude, $a_{jh}(y)$, and wave vector, $k_m(y)$, were no longer isotropic and depended on each other. In [1] there was proposed a generalized eikonal equation. Here we use the simpler and more natural variant

$$g^{ij}k_ik_j = 0. \tag{2}$$

where $k_i = \dfrac{\partial\psi}{\partial x^i}$. By a similar approach to the Riemannian case, the eikonal appears to be

$$\psi = k_i(y)x^i + h\tilde{A}(y)\sin\left(K_ix^i\right), \quad \gamma^{ij}k_ik_j = 0,\ k_i = k_i(y) \tag{3}$$

where $\tilde{A} = \tilde{A}(y)$ is expressed in terms of the wave vectors (k_i) and (K_i) of the gravitational and of the electromagnetic waves as $\tilde{A}(y) = \dfrac{1}{2}\dfrac{\tilde{a}_{ij}k^ik^j}{K_ik^i} = \dfrac{1}{2}\dfrac{\tilde{a}^{ij}k_ik_j}{\gamma^{ij}K_ik_j}$. The generalized geodesic equation took the form

$$g^*_{ij}\frac{dy^j}{dt} + \gamma_{i00} + \frac{1}{2}\varepsilon_{hl\cdot i,j}y^hy^ly^j = 0, \tag{4}$$

where $\gamma_{i00} = \gamma_{ijk}y^jy^k$; $g^*_{ij} = \dfrac{1}{2}\dfrac{\partial F^2}{\partial y^i\partial y^j}$ is the (usual) Finsler metric generated by the Finslerian function $F^2 = (\gamma_{ij}(y) + \varepsilon_{ij}(x,y))y^iy^j$ and the third term originated from the anisotropic deformation of the metric.

It should be noticed that this equation has a physical meaning. For the locally Minkowskian space with small anisotropic deformation, the force potentials consist of two terms. The second term in brackets, originating from the anisotropy of the deformation, is associated with the velocity and provides an analogue to the second term in the expression for the Lorentz force in electrodynamics. This illustrates the ideas formulated in the end of [3], [4] and developed in [8], [9].

In this paper we obtain the solutions of these equations for the flat Minkowski and Berwald-Moor metrics as unperturbed ones and use them for the calculation of the OMPR effect in Finsler case.

2 Weak anisotropic perturbation of the flat Minkowski metric

2.1 Solution

Let the initial (undeformed) metric be the flat Minkowskian one $\gamma = diag(1,-1,-1,-1)$. Then the conditions on the wave solution lead to the system

$$\begin{cases} \gamma^{hl}k_hk_l = 0 \\ a^i{}_jk_i = \dfrac{1}{2}a^i{}_ik_j \\ \dfrac{\partial}{\partial y^b}\left(\dfrac{1}{2}a^i{}_ik_j\sin(k_mx^m)\right) = \overset{0}{C}{}^i{}_{lb}\{2a^l{}_jk_i - a^l{}_ik_j\}\sin(k_mx^m). \end{cases} \tag{5}$$

obtained in [1] which is identically satisfied by

$$g_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 + h\tilde{a}(y)\cos\Big(\dfrac{D}{c}(x^1 - x^2)\Big) & 0 \\ 0 & 0 & 0 & -1 - h\tilde{a}(y)\cos\Big(\dfrac{D}{c}(x^1 - x^2)\Big) \end{pmatrix}. \tag{6}$$

Here $\tilde{a}(y)$ is an arbitrary scalar 0-homogeneous function, and $h \in \mathbb{R}$ is a constant, small enough such that $h^2 \simeq 0$. When $a(y) := h\tilde{a}(y)$ is a constant, this metric reduces to the perturbed Minkowski metric for the isotropic empty space.

In other words, the obtained perturbation is

$$\begin{aligned} \varepsilon_{33} &= h\tilde{a}(y)\cos\Big(\frac{D}{c}(x^1-x^2)\Big), \quad \varepsilon_{44} = -h\tilde{a}(y)\cos\Big(\frac{D}{c}(x^1-x^2)\Big), \\ \varepsilon_{ij} &= 0, \quad \text{for all other } (i,j). \end{aligned}$$

2.2 Eikonal

For the above perturbation of the Minkowski metric, we get $\tilde{A} = \frac{1}{2}\frac{\tilde{a}^{ij}k_ik_j}{\gamma^{ij}K_ik_j} = \frac{1}{2}\frac{c^2\widetilde{a}(y^i)(k_3^2-k_4^2)}{D(ck_2-\omega)}$, and the eikonal (3) takes the form

$$\psi = -\frac{\omega}{c}x^1 + k_2x^2 + k_3x^3 + k_4x^4 + \frac{h}{2}\frac{c^2\widetilde{a}(y^i)(k_3^2-k_4^2)}{D(ck_2-\omega)}\sin\left(K_ix^i\right), \tag{7}$$

Example: Let $v \in \mathcal{X}(M)$ be an arbitrary vector field on $M = \mathbb{R}^4$ and $\tilde{a} = \dfrac{K_iy^i}{\gamma_{ij}v^iy^j}$, where K is the wave vector. As the ratio of two invariant quantities under coordinate changes, $\tilde{a}$ is globally defined.

In the particular local frame in which $K_1 = \frac{D}{c}$, $K_2 = -\frac{D}{c}$, $K_3 = 0, K_4 = 0$ (which is, chosen such that the GW propagates antiparallel to the Ox axis), we get $\tilde{a} = \dfrac{\frac{D}{c}(y^1-y^2)}{\gamma_{ij}v^iy^j}$.

More particularly, if in the given frame, $v^i = 0$, $i = 1,2,3$ and $v^4 = -\frac{D}{c}$, then we get

$$\tilde{a} = \frac{y^1-y^2}{y^4}$$

hence,

$$\psi = -\frac{\omega}{c}x^1 + k_2x^2 + k_3x^3 + k_4x^4 + \frac{h}{2}\frac{c^2(y^1-y^2)(k_3^2-k_4^2)}{Dy^4(ck_2-\omega)}\sin\left(K_ix^i\right) \tag{8}$$

2.3 Geodesics

If $\varepsilon_{ij}(x,y) = h\tilde{a}_{ij}(y)\cos(K_m(y)x^m)$, then, performing the derivations, we obtain the geodesic equations:

$$\frac{dy^i}{ds} + hA^i(y)\sin(K_mx^m) + hB^i_p(y)x^p\cos(K_mx^m) = 0 \tag{9}$$

where the coefficients

$$\begin{aligned} A^i &= -\frac{1}{2}\gamma^{it}y^ly^s[y^j\frac{\partial(K_j\tilde{a}_{sl})}{\partial y^t} + (K_s\tilde{a}_{tl} + K_l\tilde{a}_{ts} - K_t\tilde{a}_{ls})]. \\ B^i_p &= -\frac{1}{2}\gamma^{it}y^ly^sy^jK_j\tilde{a}_{sl}\frac{\partial K_p}{\partial y^t} = -\frac{1}{2}\gamma^{it}K_0\tilde{a}_{00}\frac{\partial K_p}{\partial y^t}. \end{aligned} \tag{10}$$

depend only on the directional variables y^i. Here $\tilde{a}_{00} \equiv \tilde{a}_{nm}y^ny^m$ and $K_0 \equiv K_iy^i$. In particular, if K_i are constant, then $B^i_p = 0$, $i = 1 \div 4$ and the equations of geodesics simplify.

Solving 9, one can find that unit-speed geodesics of the perturbed metric $g_{ij}(x,y) = \gamma_{ij}(x,y) + h\tilde{a}_{ij}(y)\cos(K_m(y)x^m)$ are described by

$$x^i(s) = \alpha^i s + \beta^i - \frac{h}{2}\gamma^{it}\frac{\partial}{\partial y^t}\left(\frac{\tilde{a}_{00}}{K_0}\right)\sin(K_m x^m) - \frac{hx^p}{2}\gamma^{it}\frac{\tilde{a}_{00}}{K_0}\frac{\partial K_p}{\partial y^t}\cos(K_m x^m), \tag{11}$$

where α^i and β^i depend on the initial conditions. In particular, if K_m are constant, geodesics of the perturbed metric obey

$$x^i(s) = \alpha^i s + \beta^i - \frac{h}{2}\gamma^{it}\frac{\partial}{\partial y^t}\left(\frac{\tilde{a}_{00}}{K_0}\right)\sin(K_m x^m). \tag{12}$$

From (11), we get that along geodesics $hx^i(s) \simeq h(\alpha^i s + \beta^i)$, and $hy^i(s) = h\frac{dx^i}{ds} \simeq h\alpha^i$. Eq. (12) describes the geodesics in the case of a small anisotropic perturbation of the Minkowski metric.

Examples: 1) For $a = h = const$ (Riemannian perturbation), that is, $\tilde{a} = 1$, we get the expression obtained in [2].

2) If $a = h\frac{y^1 - y^2}{y^4}$ as earlier, we get $a = \frac{h\alpha}{y^4}$ along geodesics, and, with $x^i(0) = 0$, $i = \overline{1,4}$ ($\Rightarrow \beta = \nu = 0$),

$$x^3 = u_0\frac{x^1 - x^2}{h\alpha} + u_0\frac{hc}{y^4 D}\sin\left(\frac{D}{c}(x^1 - x^2)\right). \tag{13}$$

Then the Oy-component of the atom velocity will contain a term proportional to

$$y^3 \sim u_0^2\frac{h\alpha}{y^4}\cos\left(\frac{D}{c}(x^1 - x^2)\right)$$

and the amplitude factor in front of the cosine depends on the velocity component, y^4, orthogonal to Ox and Oy axes.

3 Weak perturbation of the anisotropic Berwald-Moor metric

3.1 Solution

Instead of the anisotropic correction to the isotropic (Minkowski) metric, we could try an originally anisotropic but still locally Minkowskian (i.e. spatial variables independent) metric on $\mathbf{R}^4$. Let us consider the Finslerian Berwald-Moor metric $\gamma_{ij}(y) = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$, in which $F = \sqrt[4]{y^1 y^2 y^3 y^4}$. The explicit form of the unperturbed metric is provided by the matrices

$$(\gamma_{ij}) = \frac{F^2}{8}\begin{pmatrix} \frac{-1}{(y^1)^2} & \frac{1}{y^1y^2} & \frac{1}{y^1y^3} & \frac{1}{y^1y^4} \\ \frac{1}{y^1y^2} & \frac{-1}{(y^2)^2} & \frac{1}{y^2y^3} & \frac{1}{y^2y^4} \\ \frac{1}{y^1y^3} & \frac{1}{y^2y^3} & \frac{-1}{(y^3)^2} & \frac{1}{y^3y^4} \\ \frac{1}{y^1y^4} & \frac{1}{y^2y^4} & \frac{1}{y^3y^4} & \frac{-1}{(y^4)^2} \end{pmatrix}, \quad (\gamma^{ij}) = \frac{2}{F^2}\begin{pmatrix} -(y^1)^2 & y^1y^2 & y^1y^3 & y^1y^4 \\ y^1y^2 & -(y^2)^2 & y^2y^3 & y^2y^4 \\ y^1y^3 & y^2y^3 & -(y^3)^2 & y^3y^4 \\ y^1y^4 & y^2y^4 & y^3y^4 & -(y^4)^2 \end{pmatrix}. \tag{14}$$

The wave solutions $\varepsilon_{ij} = a_{ij}(y)\cos(K_m x^m)$ for Einstein's equations in vacuum are given by the solutions of the system (5), where the coefficients, $\overset{0}{C}{}^i{}_{jd} = \frac{1}{2}\gamma^{ih}\frac{\partial\gamma_{hj}}{\partial y^d}$ will be given by

$$\overset{0}{C}{}^i{}_{jd} = \frac{p}{8}\frac{y^i}{y^j y^d}, \quad p = \begin{cases} -\frac{3}{8}, & \text{if} \quad i=j=d \\ \frac{1}{8}, & \text{if} \quad i=j\neq d \ \text{ or } i\neq j=d \ \text{ or } \ i=d\neq j. \\ -\frac{1}{8}, & \text{if} \quad i\neq j\neq d\neq i. \end{cases} \tag{15}$$

If we choose the coordinate system such that

$$K_3 = K_4 = 0, \tag{16}$$

then the light-like condition $\gamma^{ij}K_iK_j = 0$ leads to

$$K_2 = \frac{y^1}{y^2}K_1. \tag{17}$$

Moreover, $a_{ij} = h\lambda(y)K_iK_j$ (here $\lambda(y)$ is an arbitrary scalar 0-homogeneous function and h is a small constant $h^2 \simeq 0$) defines a solution of (5) obeying the transverse traceless conditions $a^i{}_i = 0, \quad a^i{}_jK_i = 0$. We get

Proposition 1 *The following perturbation defines a solution for the Einstein equations in vacuum for the Bewald-Moor metric:*

$$\varepsilon_{ij}(x,y) = h\lambda\tilde{a}_{ij}\cos(K_1x^1 + K_2x^2), \tag{18}$$

$$(\tilde{a}_{ij}) = \begin{pmatrix} K_1^2 & K_1K_2 & 0 & 0 \\ K_1K_2 & K_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{19}$$

where the first component $K_1 = K_1(y)$ of the wave vector is an arbitrary 0-homogeneous function of the directional variables and K_2 obeys relation 17.

Let us denote, in some fixed coordinate system:

$$K_i = \frac{D}{y^i}, \qquad n_i = \frac{c}{D}K_i, \qquad\qquad i = 1,...,4.$$

With these, the solution can be written as:

$$\varepsilon_{ij} = h\frac{\lambda D^2}{c^2}n_in_j\cos(\frac{D}{c}(n_ix^i)). \tag{20}$$

In the given frame, we have $K_2 = \frac{D}{y^2}$, $n_3 = n_4 = 0$.

Example:

An example which is interesting because of its symmetry, is $\lambda D^2 = y^1y^2$. Then,

$$(\tilde{a}_{ij}) = \lambda D^2 \begin{pmatrix} \frac{1}{(y^1)^2} & \frac{1}{y^1y^2} & 0 & 0 \\ \frac{1}{y^1y^2} & \frac{1}{(y^2)^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = h\begin{pmatrix} \frac{n_1}{n_2} & 1 & 0 & 0 \\ 1 & \frac{n_2}{n_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}; \tag{21}$$

and we see that in this case the perturbation, ε_{ij}, of the metric becomes

$$\varepsilon_{ij} = h\frac{n_i n_j}{n_1 n_2}\cos[\frac{D}{c}(n_1 x^1 + n_2 x^2)], \quad i,j = 1,2. \tag{22}$$

3.2 Eikonal equation

With the help of $\tilde{A} = \frac{1}{2}\frac{\tilde{a}^{ij}k_i k_j}{\gamma^{ij}K_i k_j} = \frac{1}{2}\frac{\lambda(K^i k_i)^2}{K^i k_i} = \frac{\lambda(y)}{2}K^i k_i$, eq. (2) yields the solution for the eikonal (3). Rewriting $\tilde{A} = \frac{\lambda D}{2c}n_i k^i$, one obtains the solution for the eikonal as:

$$\psi = k_i(y)x^i + h\frac{\lambda D}{2c}n_i k^i \sin\left[\frac{D}{c}\left(n_1 x^1 + n_2 x^2\right)\right] \tag{23}$$

Example: for $\lambda D^2 = y^1 y^2$, we get

$$\psi = k_i(y)x^i + h\frac{c}{2D}\frac{(n_1 k^1 + n_2 k^2)}{n_1 n_2}\sin\left[\frac{D}{c}\left(n_1 x^1 + n_2 x^2\right)\right]$$

Equation (23) describes the eikonal of the wave propagating in the model anisotropic space-time with the Berwald-Moor metric perturbed by the GW. In the Berwald-Moor case, the components $k_i(y)$ cannot be constant, since the equation $\gamma^{ij}k_i k_j = 0$ does not have any constant solutions (except the trivial one $k_i = 0, i = 1, ..., 4$).

3.3 Geodesics

Equations (4), for linearized perturbations of BM metric, lead again to the equivalent formulation (9), (10) and to the solution (11). In the case under discussion we get $K_0 = K_1 y^1 + K_2 y^2 = 2D$, $\tilde{a}_{00} = \lambda K_0^2 = 4\lambda D^2$, $\frac{\tilde{a}_{00}}{K_0} = 2\lambda D$, hence unit-speed geodesics ($F = 1$) obey

$$x^i(s) = \alpha^i s + \beta^i - h\gamma^{ij}\frac{\partial(\lambda D)}{\partial y^j}\sin(K_m x^m) - h\gamma^{ij}\frac{\partial K_p}{\partial y^j}x^p \lambda D\cos(K_m x^m). \tag{24}$$

The simplest solutions are obtained for

$$D := D(y^1, y^2);$$

in this case, performing the calculations, we find $s = \frac{K_1 x^1 + K_2 x^2}{K_1 \alpha^1 + K_2 \alpha^2}$. In this case, the cosine term in (24) vanishes in the expressions of x^3 and x^4. Calculating the derivative and using the initial conditions, $x^i(0) = 0 \Rightarrow \beta^i = 0$, $i = 1,2,3$, we get the Oy-component of the velocity

$$y^3 = \alpha^3 - 4\frac{h\lambda D^2 y^3}{F^2}\cos(K_m x^m). \tag{25}$$

Example: if $\lambda D^2 = y^1 y^2$, then

$$y^3 = \alpha^3 - 4\frac{hF^2}{y^4}\cos(K_m x^m) \overset{F^2=1}{=} \alpha^3 - 4h\frac{1}{y^4}\cos(K_m x^m) \tag{26}$$

4 OMPR modifications

4.1 Flat Minkowski space with weak anisotropic perturbation

The physical interpretation of the obtained solutions leading to the modifications in the OMPR effect is the following. One can see that the anisotropy does not destroy the solution of the OMPR equations given in [1]. For a simple anisotropic deformation of the Minkowski metric, we get the dependence of eqs. (8, 13) on the directional variable orthogonal to Ox and Oy, i.e. to the plane containing the Earth, the space maser and the GW source. This plane can belong to the galaxy plane and can be perpendicular to it.

Geodesics describe the trajectory of the particle, and the sample eq. (13) means that the amplitude of the oscillations of the space maser atom velocity component oriented at the Earth, y^3, depends on y^4. This means that when the system "Earth-space maser-GW source" is located close to the periphery of the galaxy, the orientation of this system might affect the OMPR conditions. In our example the OMPR conditions must be modified and take the form

$$\frac{\alpha_2}{\alpha_1} = \frac{\omega h}{8Dy^4} = b\varepsilon; b = O(1); \varepsilon << 1 \tag{27}$$

$$\frac{kv_1}{\alpha_1} = \frac{\omega hc}{\alpha_1 y^4} = \kappa\varepsilon; \kappa = O(1); \varepsilon << 1 \tag{28}$$

that illustrates the qualitative analysis given in [4]. This means that the experimental investigation of the astrophysical systems with various orientations might provide the information on the quantitative characteristics of the geometrical anisotropy (if any) of our galaxy.

4.2 Anisotropic space with Berwald-Moor metric and weak perturbation

As in the previous case, the anisotropy does not destroy the OMPR effect itself, but now the modifications are more pronounced. Eq.(23) for the eikonal also gives a trichromatic EMW, but the amplitudes of the sidebands and their frequencies are now different from the isotropic case. The geodesics in the form (25) shows that the amplitude of the atomic oscillations is now also different. All this would affect the OMPR conditions and they would be modified in the following way

$$\frac{\alpha_2}{\alpha_1} = h\frac{\lambda D}{4c}n_i k^i = b\varepsilon; \quad b = O(1); \quad \varepsilon \ll 1 \tag{29}$$

$$4h\frac{\omega}{\alpha_1}\frac{\lambda D^2}{c^2}\sqrt{\frac{n_1 n_2 n_4}{n_3}} = \kappa\varepsilon; \quad \kappa = O(1); \quad \varepsilon \ll 1 \tag{30}$$

$$(\omega - \Omega + kv_0)^2 + 4\alpha_1^2 = D^2 n_1^2 + O(\varepsilon) \Rightarrow Dn_1 \sim 2\alpha_1 \tag{31}$$

or, for the sample example, $\lambda D^2 = y^1 y^2$,

$$\frac{\alpha_2}{\alpha_1} = h\frac{c}{4D}\frac{(n_1 k^1 + n_2 k^2)}{n_1 n_2} = b\varepsilon; \quad b = O(1); \quad \varepsilon \ll 1 \tag{32}$$

$$4h\frac{\omega}{\alpha_1}\sqrt{\frac{n_4}{n_1 n_2 n_3}} = \kappa\varepsilon; \quad \kappa = O(1); \quad \varepsilon \ll 1 \tag{33}$$

$$(\omega - \Omega + kv_0)^2 + 4\alpha_1^2 = D^2 n_1^2 + O(\varepsilon) \Rightarrow Dn_1 \sim 2\alpha_1 \tag{34}$$

As in the previous Section, we find that the orientation of the system would affect the observations. Calculating the left hand sides of the second condition in (32) for the systems belonging and perpendicular to the galactic plane, one can see that their ratio is equal to the ratio of the star velocity corresponding to the galactic rotation and the star velocity in the direction of the galaxy axis. Therefore, if we take two equivalent astrophysical systems that initially suffice the OMPR conditions and differ only by their orientation with regard to the galactic plane, only one of them will produce an observable OMPR signal.

5 Discussion

The main results obtained in this paper are the following. In search for the modifications of the Einstein-Hilbert action due to the anisotropy of the space-time, we have constructed two simple models of the anisotropic space-time with metrics containing small perturbations. The additional terms lead to a change of the OMPR conditions in the anisotropic space-time. It turned out that the orientation of the astrophysical system (taking part in the OMPR) with regard to the galactic plane causes changes in the observable effect, thus, giving one the possibility to experimentally investigate the space-time geometrical properties on the galactic scale.

The expression for the "simplest scalar" which can be used in the variation principle based on the Einstein-Hilbert expression for the action was particularized for our model. If the perturbed locally Minkowskian metric can be presented as $g_{ij}(x,y) = \gamma_{ij}(y) + \varepsilon_{ij}(x,y)$, then the space-time anisotropy produces additional terms to the usual Ricci tensor $R_{\ jk}$ which is to be calculated with regard to $\Gamma^i_{\ jk}$ equal to

$$\Gamma^i_{\ jk} = \frac{1}{2}\gamma^{il}\left(\frac{\partial\varepsilon_{lj}}{\partial x^k} + \frac{\partial\varepsilon_{lk}}{\partial x^j} - \frac{\partial\varepsilon_{jk}}{\partial x^l}\right) = -\frac{1}{2}\gamma^{il}(a_{lj}K_k + a_{lk}K_j - a_{jk}K_l)\sin(K_m x^m), \tag{35}$$

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